Algebra of physical space and the geometric spacetime solution of Dirac's equation

Burke Ritchie
Lawrence Livermore National Laboratory and
Livermore Software Technology Corporation
Livermore, California
and
Charles A. Weatherford
Physics Department
Florida A&M University
Tallahassee, Florida

The algebra of physical space (APS) is a name for the Clifford or geometric algebra which can be closely associated with the geometry of special relativity and relativistic spacetime. For example the Dirac Hamiltonian can be presented as the scalar product of the electron's 4-momentum and Dirac's 4-vector of gamma matrices, $(\gamma_0,\vec{\gamma})$, the latter of which is a Clifford algebra. We show here that a geometric spacetime or 4-space solution of Dirac's equation conforms to the principles of APS, an early example of which may be Schroedinger's solution of Dirac's equation for a free electron, which exhibits Zitterbewegung. In a 4-space solution the spacetime coordinates, $\vec{r}$ and the scaled time ct, are treated on an equal footing as physical observables in order to avoid any suggestion of a preferred frame of reference.

The geometric spacetime theory is studied here for the Coulomb problem. The positive-energy spectrum of states is found to be identical within numerical error to that of standard Dirac theory, but the wavefunction requires a modified interpretation. It is shown analytically how the geometric spacetime solution can be reduced to the standard solution of Dirac's equation. The rigor of APS and of its conforming geometric spacetime solution provide strong support for further investigation into the physical interpretation of the geometric spacetime Dirac wave function.