

# Topological Index and Homotopy in Coupled-Cluster theory

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We propose a comprehensive mathematical framework for Coupled-Cluster-type methods based on topological degree theory [1]. This allows us to establish more general existence results than [10] and deduce local information about the solutions of the CC equations. The idea of constructing a homotopy for CC theory is not new, and has been extensively studied in the past [11, 12, 8, 5, 7, 4, 2, 3, 6]. We consider the more recent Kowalski–Piecuch (KP) homotopy [9] from a mathematical point of view and use it as a theoretical tool to prove the existence of a truncated CC solution. This follows from a more general result guaranteeing the existence of a whole solution curve of the KP homotopy.

## References

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